The Quark-Meson Coupling model as a description of dense matter

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Outline

1. The Basics:
   - The Nature of Dense Matter
   - The Models
2. Simulations:
   - Hadronic Matter
   - Mixed-Phase Matter

Motivation

We wish to understand the properties of matter over a wide range of densities; from single atomic nuclei to neutron stars.

In order to do this, we must first understand the fundamental and effective degrees of freedom at each density scale.

In order to evaluate success, we need to compare predictions over a wide range of density scales to observable phenomena.

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- Quarks and Gluons are the fundamental degrees of freedom
- At low densities, Baryons (Nucleons) are the effective degrees of freedom
- At high densities...

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- At high densities... ??? = Hyperons? Quarks? Other?

Hadronic Models
A Brief Overview

Quantum **H**adro**D**ynamics (**QHD**) Model

- Simple description of nucleons immersed in mean-field σ, ω, and ρ potentials,
- Constructed at the baryon level,
- Issues with large scalar potentials causing negative effective masses.

Quark-**M**eson **C**oupling (**QMC**) Model

- Similar final form as QHD, but with self-consistent response to the σ field, despite construction from quark level,
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$$M_B^* = M_B + \Sigma_B^s = M_B - w_B^s g_{\sigma N}\langle\sigma\rangle + \frac{d}{2}\tilde{w}_B^s\left(g_{\sigma N}\langle\sigma\rangle\right)^2$$

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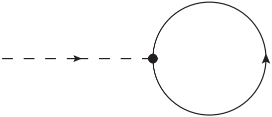
In QHD, at Hartree level (mean-field), the scalar self-energy involves only the tadpole term:



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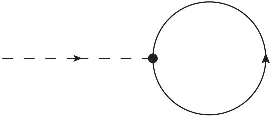

Hartree Σ_B^s (QHD)

$$\begin{aligned}
\Sigma_B^s &= -g_{\sigma B}\langle\sigma\rangle \\
&= -g_{\sigma B}\sum_{B'}\frac{g_{\sigma B'}}{m_\sigma^2}\frac{(2J_{B'}+1)}{(2\pi)^3}M_{B'}^*\int\frac{\theta(k_{F_{B'}}-|\vec{k}|)\,d^3k}{\sqrt{\vec{k}^2+M_{B'}^{*2}}}
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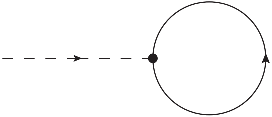

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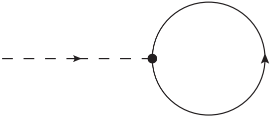

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Hyperonic QMC

- $B \in \{p, n, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0\} = \{N, Y\}$
- $\ell \in \{e^-, \mu^-\}$
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Saturation

$$\left.\begin{aligned} (E/A)_{\rho_0} &= -15.86 \text{ MeV}, \\ (\rho_{\text{total}})_{\rho_0} &= 0.16 \text{ fm}^{-3} \end{aligned}\right\} \quad -- \quad g_{\sigma N},\ g_{\omega N}$$

$$(a_{sym})_{\rho_0} = 32.5 \text{ Mev} \quad -- \quad g_{\rho N}$$

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Effective masses from Ref. [4]: Guichon *et. al.* doi:10.1016/j.nuclphysa.2008.10.001 (previously from Ref. [5]: Rikovska-Stone *et. al.* doi:10.1016/j.nuclphysa.2007.05.011) derived from the bag model.

Hyperonic QMC

Equation of State (EOS) is calculated assuming that

Chemical Equilibrium

$$\mu_i = B_i\,\mu_n - Q_i\,\mu_e = \sqrt{k_{F_i}^2 + (M_i + \Sigma_i^s)^2} + \Sigma_i^0$$

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Hadronic Models
A Brief Overview: Finite Nuclei

Finite Nuclei:

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A Brief Overview: Finite Nuclei

The mean-fields $\langle m \rangle$ are calculated via the equations of motion;

Equations of Motion

$$\begin{aligned} \left(\Box + m_\sigma^2\right)\sigma &= g_{N\sigma}\bar{\psi}\psi, \\ \partial^\mu \Omega_{\mu\nu} &= g_{N\omega}\bar{\psi}\gamma_\nu\psi - m_\omega^2\omega_\nu, \\ \partial^\mu R^a_{\mu\nu} &= g_\rho\bar{\psi}\gamma_\nu\tau^a\psi - m_\rho^2\rho^a_\nu. \end{aligned}$$

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\left(-\nabla^2 + m_\sigma^2\right) \sigma(x) &= -g_{N\sigma} \mathrm{Tr}[iG_H(x,x)], \\
\left(-\nabla^2 + m_\omega^2\right) \omega^\mu(x) &= -g_{N\omega} \mathrm{Tr}[i\gamma^\mu G_H(x,x)], \\
\left(-\nabla^2 + m_\rho^2\right) \rho^{\mu a}(x) &= -g_\rho \mathrm{Tr}[i\tau^a \gamma^\mu G_H(x,x)]
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Consider the solutions of the Dirac equation to be written as

Dirac Solutions

$$U_\alpha(x) = U_{n\kappa mt}(x) = \begin{bmatrix} iG_{n\kappa t}(r)/r & \Phi_{\kappa m}\eta_t \\ -F_{n\kappa t}(r)/r & \Phi_{-\kappa m}\eta_t \end{bmatrix}$$

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Equations of Motion

$$\frac{d^2}{dr^2}\sigma_0(r) + \frac{2}{r}\frac{d}{dr}\sigma_0(r) - m_\sigma^2\sigma_0(r) = -g_{N\sigma}\sum_{\alpha}^{\rm occ}\left(\frac{2j_\alpha+1}{4\pi r^2}\right)\left(|G_\alpha(r)|^2 - |F_\alpha(r)|^2\right),$$

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- $\Rightarrow$ $\rho_p(r)$, $\rho_n(r)$, $\rho_B(r)$
- ϵ_α, masses/splittings
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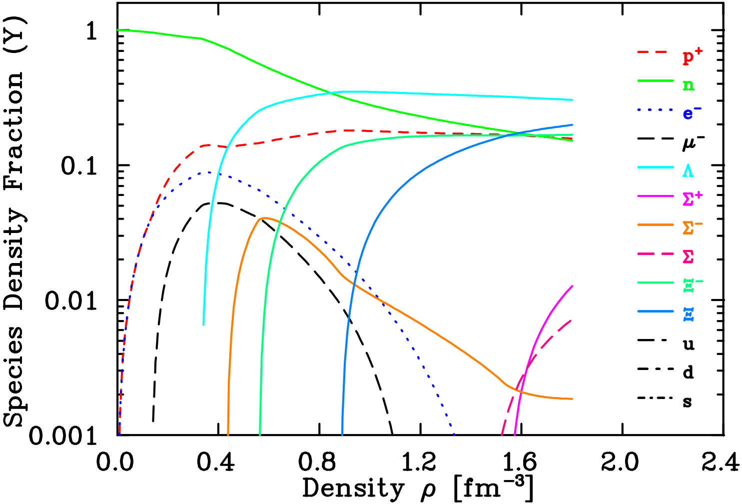

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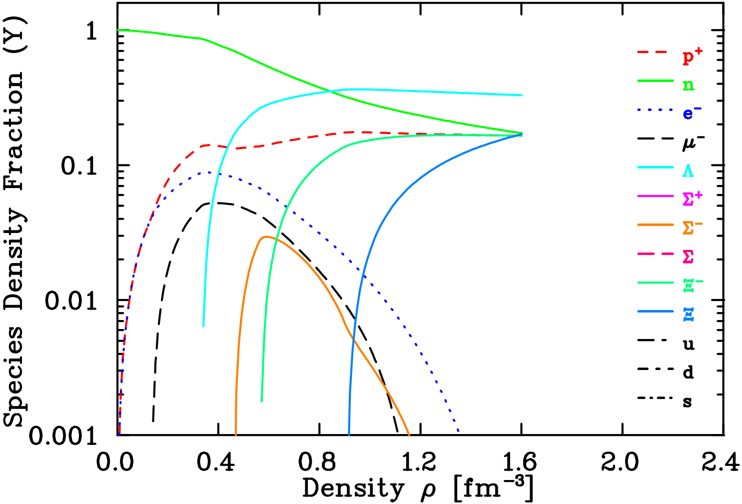


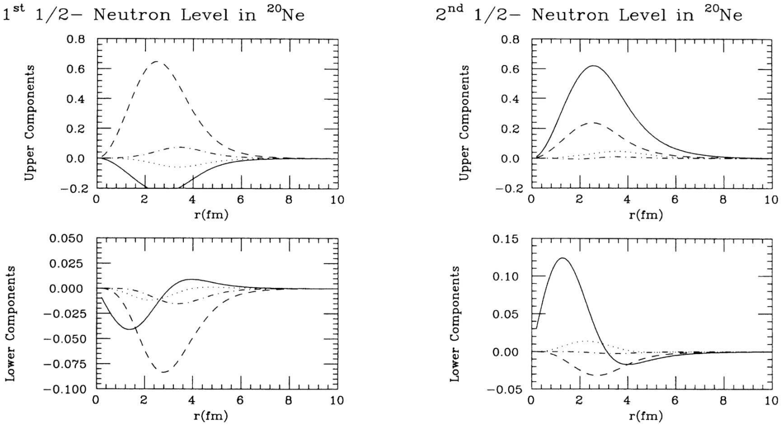
Interpretation

The improvement in the 2008 parameterization of M^* is that the effect of the mean scalar field in-medium on the familiar one-gluon-exchange hyperfine interaction (that in free space leads to the N-Δ and Σ-Λ mass splittings) is also included self-consistently.

This has the effect of increasing the splitting between the Λ and Σ masses as the density rises and the prime reason why we find that the Σ hypernuclei are unbound.

Guichon, Thomas, Tsushima: 2008

QMC - Finite Nuclei



[e.g. from C. E. Price, G. E. Walker: Phys.Rev.C36:354-364 (1987)]

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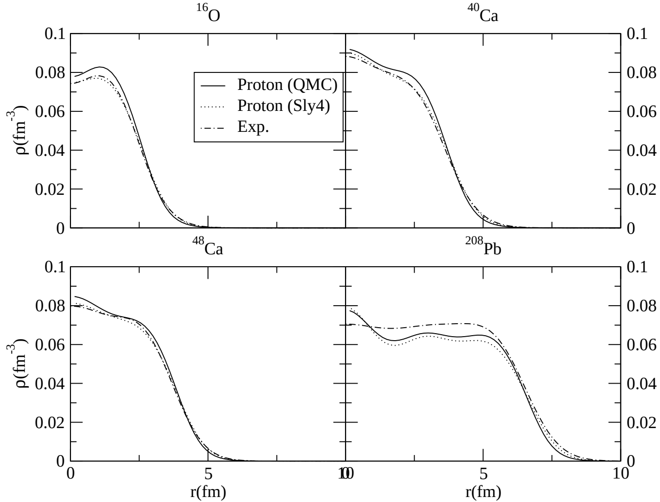

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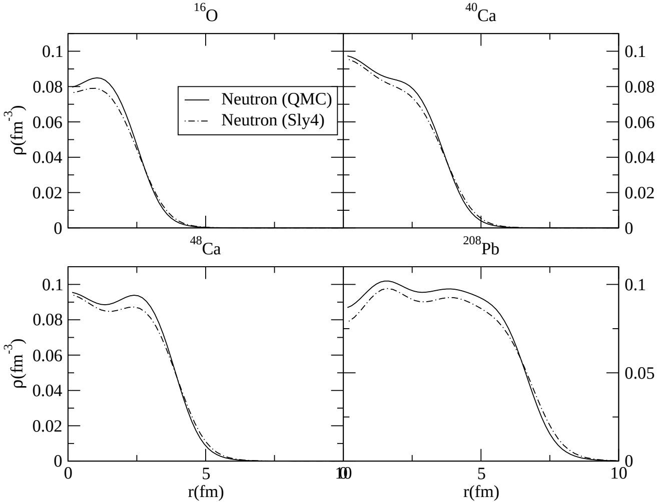

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QMC - Finite Nuclei

	E_B (MeV) [experiment]	E_B (MeV) [QMC]	r_c (fm) [experiment]	r_c (fm) [QMC]
^{16}O	7.976	7.618	2.73	2.702
^{40}Ca	8.551	8.213	3.485	3.415
^{48}Ca	8.666	8.343	3.484	3.468
^{208}Pb	7.867	7.515	5.5	5.42

[from H. H. Matevosyan: Ph.D. Thesis (2007)]

Tolman-Oppenheimer-Volkoff

Equations describe a static, spherically symmetric, non-rotating star, stable against gravitational collapse;

TOV

$$\frac{dP}{dr} = -\frac{G\left(P/c^2 + \mathcal{E}\right)\left(M(r) + 4r^3\pi P/c^2\right)}{r(r - 2GM(r)/c^2)}$$

$$M(R) = \int_0^R 4\pi r^2 \mathcal{E}(r)\, dr$$

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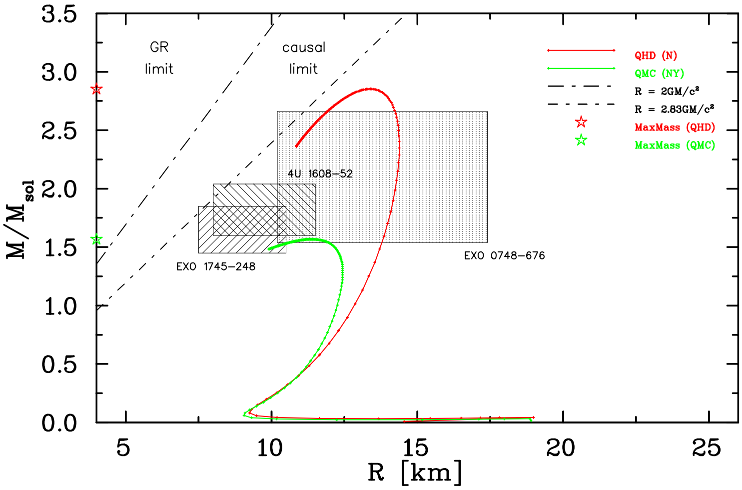
Hyperonic QMC
TOV solutions



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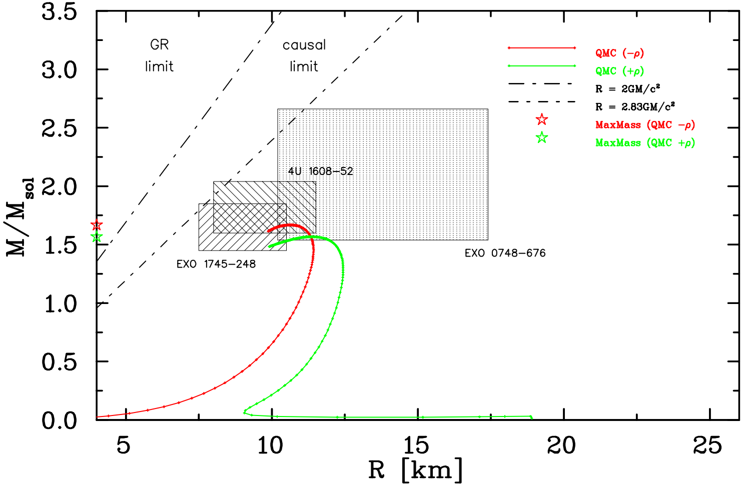

What about higher densities?

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Quark Models
A Brief Overview

MIT Bag Model

- 3 quarks in a '*bag*',
- Separated from the QCD vacuum by an energy-density B,
- Constant, current-quark masses

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- Simple inclusion of DχSB,
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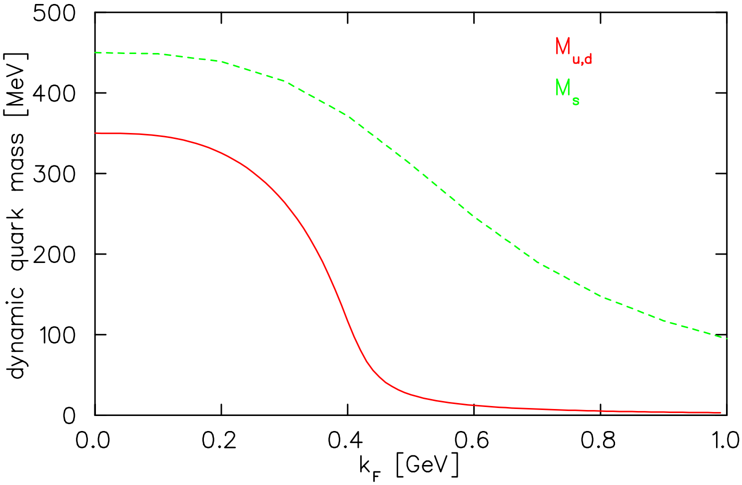
$$\begin{aligned} k_F = 0 \quad &: \quad m_u = 350\ \text{MeV},\ m_d = 350\ \text{MeV},\ m_s = 450\ \text{MeV} \\ k_F = \Lambda \quad &: \quad m_u = 5\ \text{MeV},\ m_d = 7\ \text{MeV},\ m_s = 95\ \text{MeV} \end{aligned}$$

Quark Models

NJL Effective Masses



Phase Transitions

The Gibbs Conditions for a phase transition are;

Gibbs Conditions

- $T_H = T_Q$ – Thermal Equilibrium
- $(\mu_i)_H = (\mu_i)_Q$ – Chemical Equilibrium
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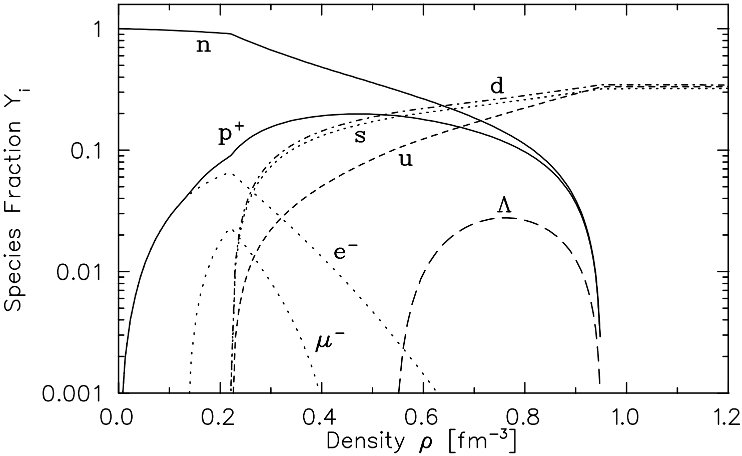

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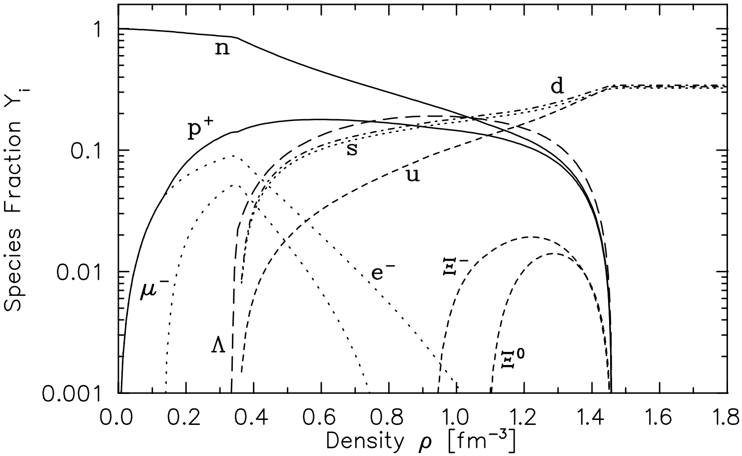

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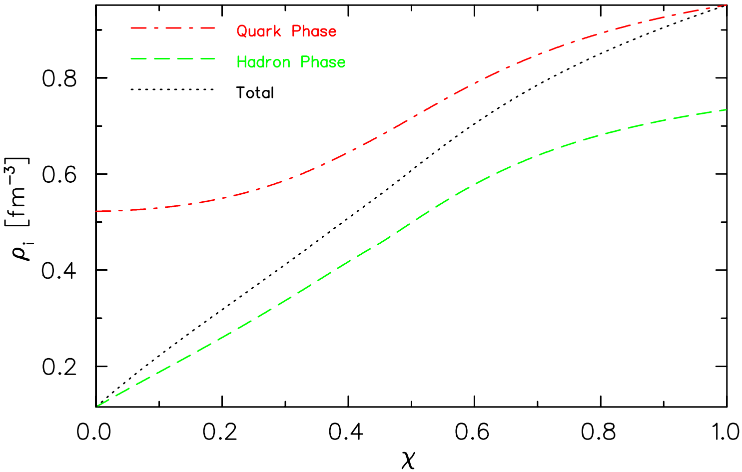

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Quark Chemical Potentials related to independent chemical potentials;

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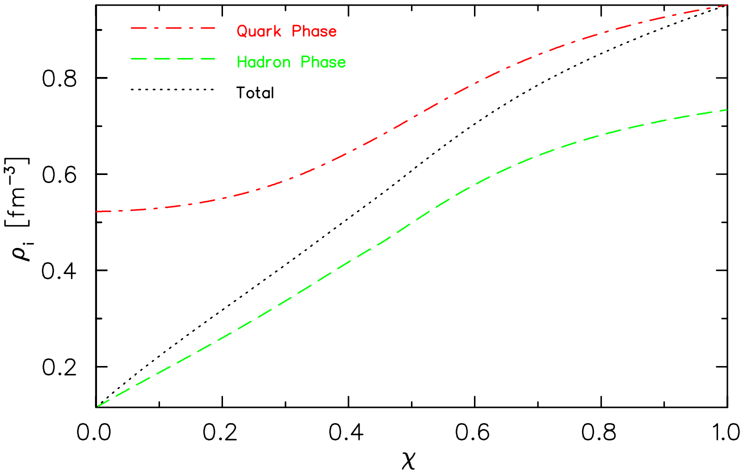

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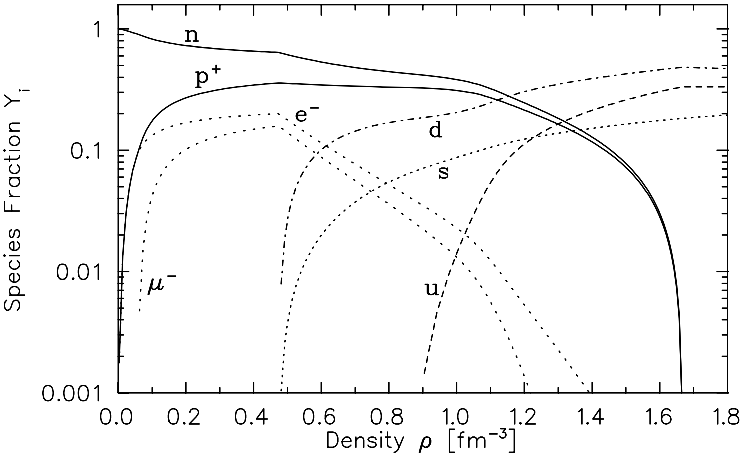
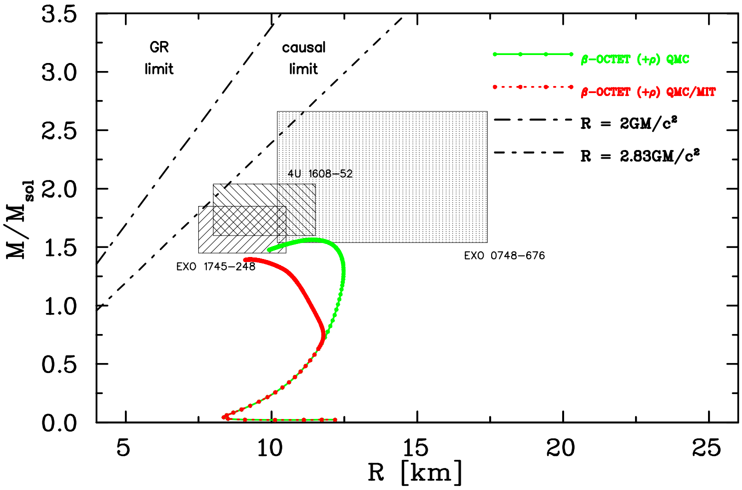


Mixed-Phase Hyperonic QMC

TOV solutions



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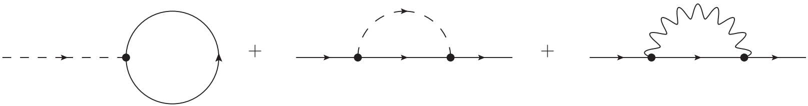
A Brief Overview: Hartree–Fock

At Hartree–Fock level, the scalar self-energy also includes an exchange term, and becomes momentum-dependent:



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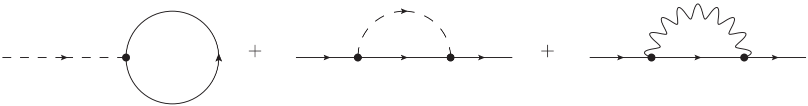

Hartree–Fock $\Sigma^s_B(k)$ (QHD)

$$\begin{aligned}
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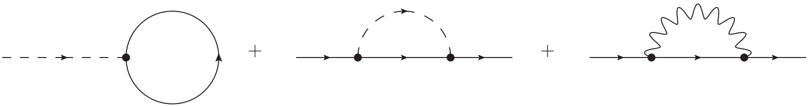

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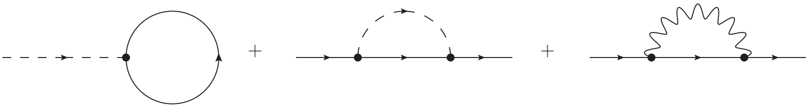

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OR... $m = \langle m \rangle + \delta m$

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Hartree–Fock H_s (QHD)

$$H_\sigma = \int d\vec{r} \left[E(\langle \sigma \rangle) - \frac{1}{2}\langle \sigma \rangle \left\langle \frac{\partial E}{\partial \langle \sigma \rangle} \right\rangle + \frac{1}{2}\delta\sigma \left(\frac{\partial E}{\partial \langle \sigma \rangle} - \left\langle \frac{\partial E}{\partial \langle \sigma \rangle} \right\rangle \right) \right]$$

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Further Reading I

Carroll
Applications of the Octet Baryon Quark-Meson Coupling Model to Hybrid Stars (PhD Thesis).
arXiv:1001.4318

Carroll, Thomas
The Hyperfine, Hyperonic QMC Model - Extension to Hartree–Fock I: Infinite Nuclear Matter.
in preparation

Carroll, Leinweber, Williams, Thomas
Phase Transition from QMC Hyperonic Matter to Deconfined Quark Matter.
Phys.Rev.C79:045810, 2009
[doi:10.1103/PhysRevC.79.045810]

Further Reading II

Guichon, Thomas, Tsushima
Binding of hypernuclei in the latest quark-meson coupling model.
Nucl.Phys.A814:66-73, 2008
[doi:10.1016/j.nuclphysa.2008.10.001]

Rikovska-Stone, Guichon, Matevosyan, Thomas
Cold uniform matter and neutron stars in the quark-mesons-coupling model.
Nucl.Phys.A792:341-369, 2007
[doi:10.1016/j.nuclphysa.2007.05.011]

Thank You!

$\mathcal{F}in$